Eventually safe languages

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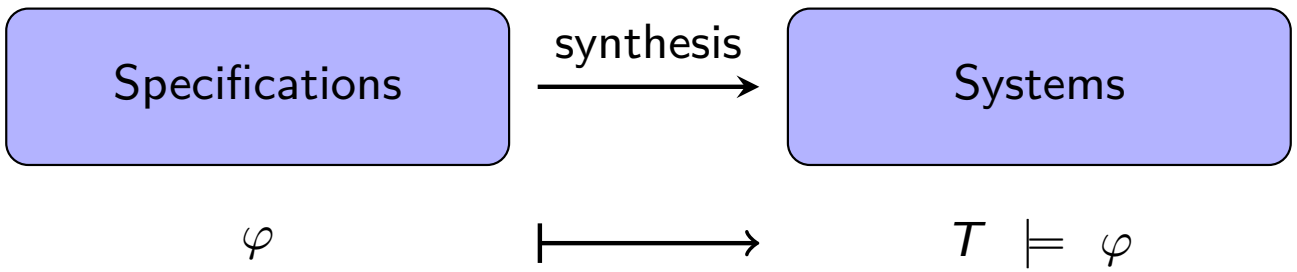
Plan

Synthesis problem (Church)



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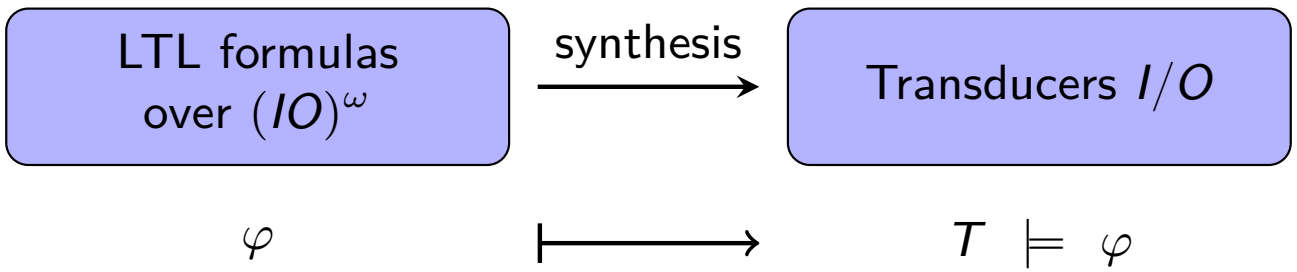

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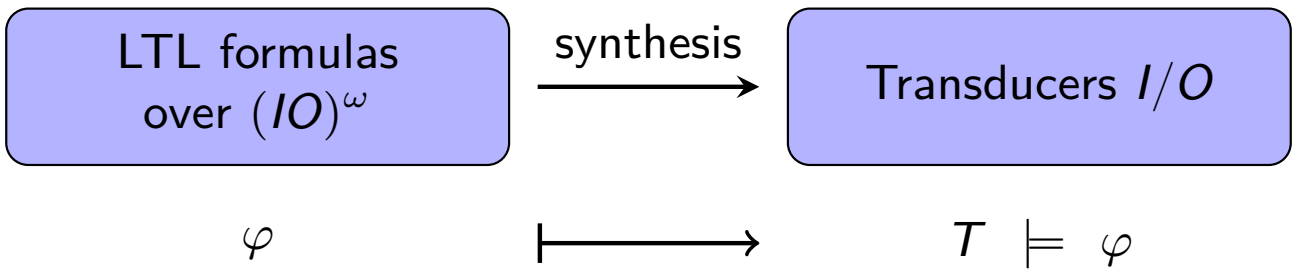

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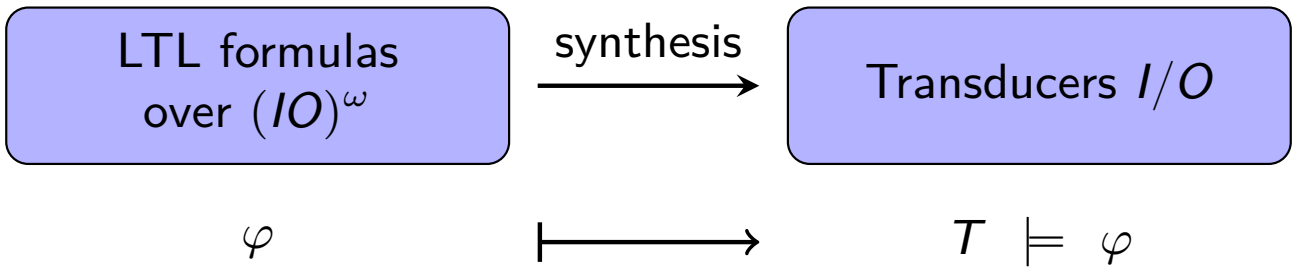

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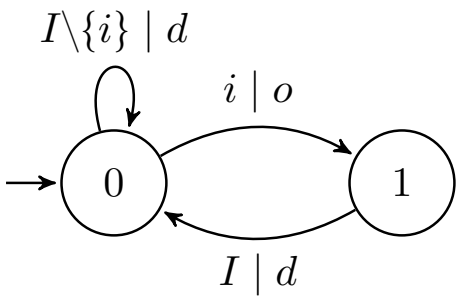

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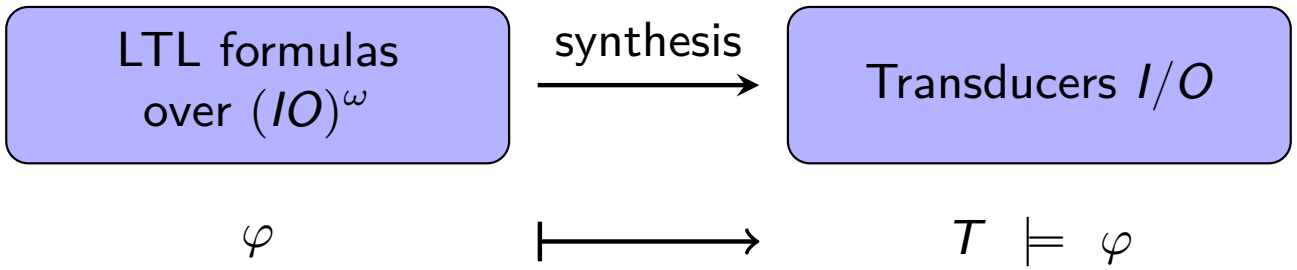

Standard solution (Büchi, Landweber)

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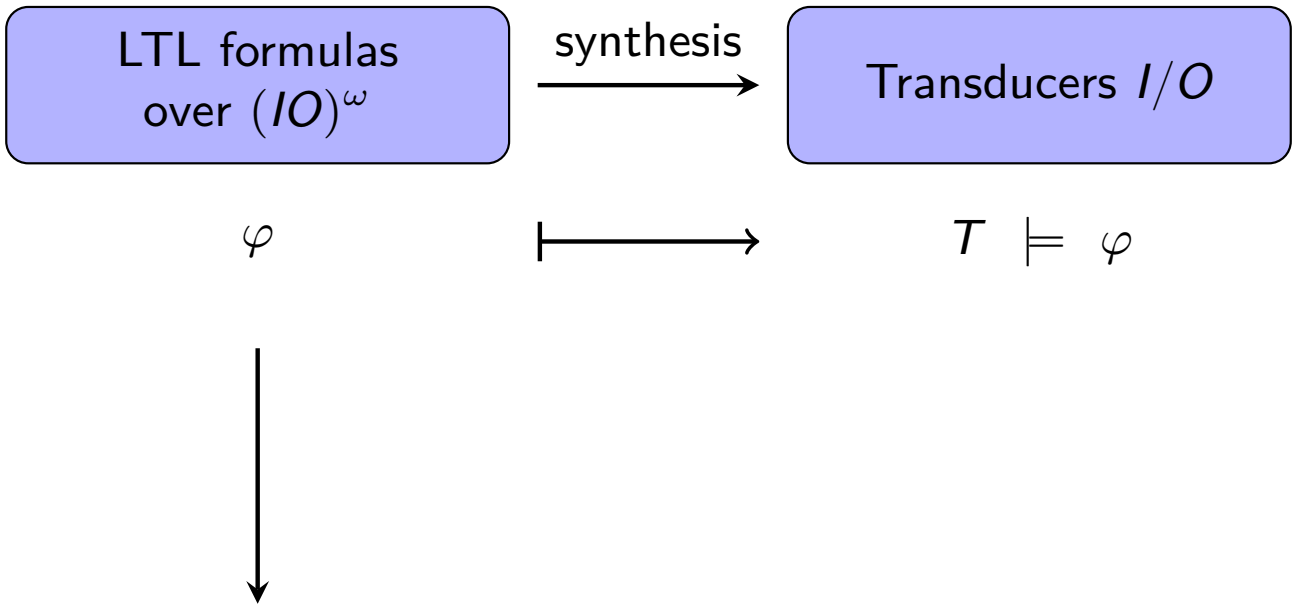

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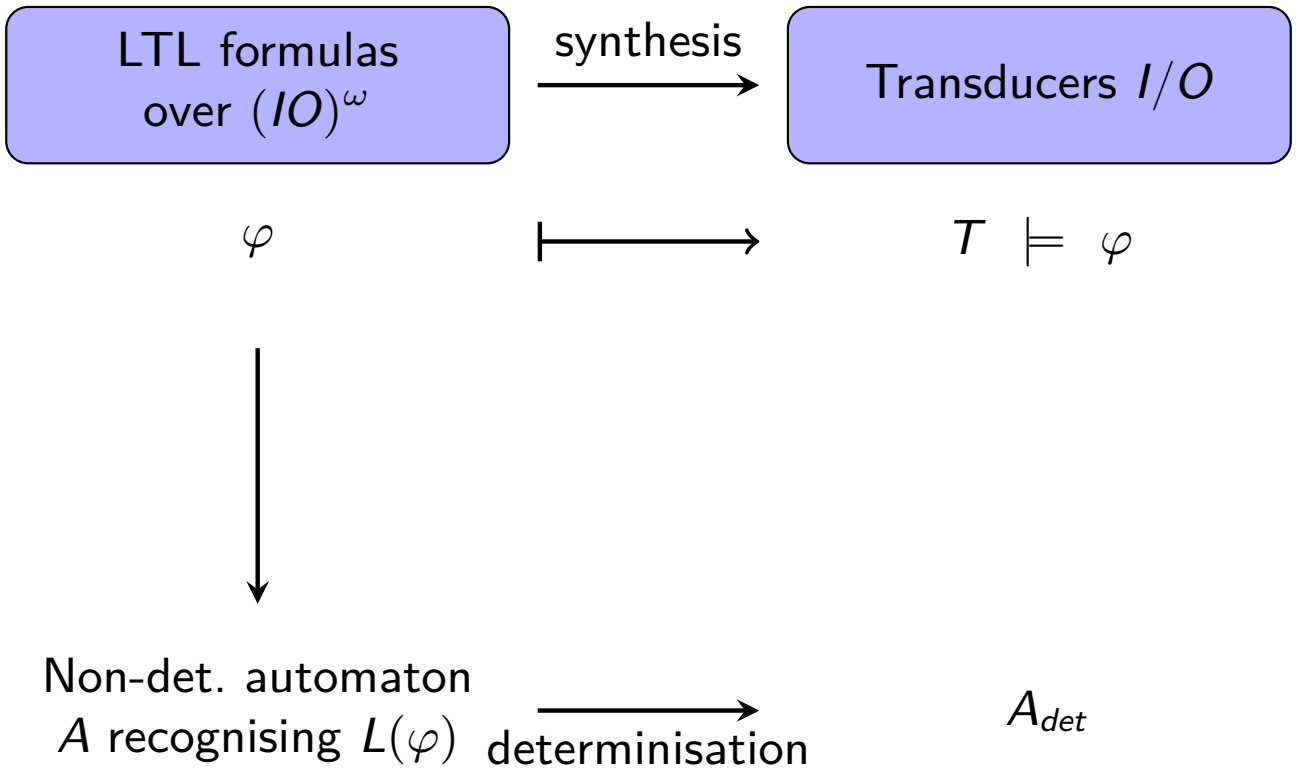

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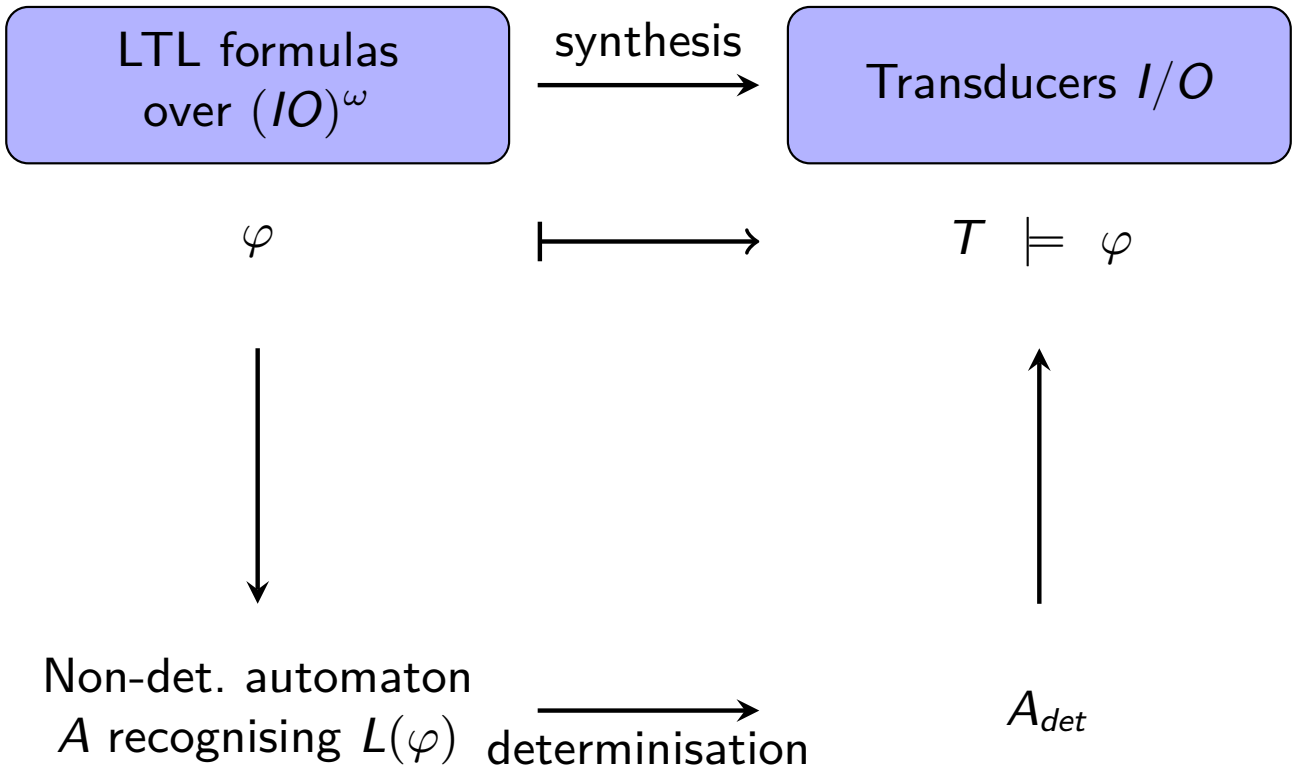

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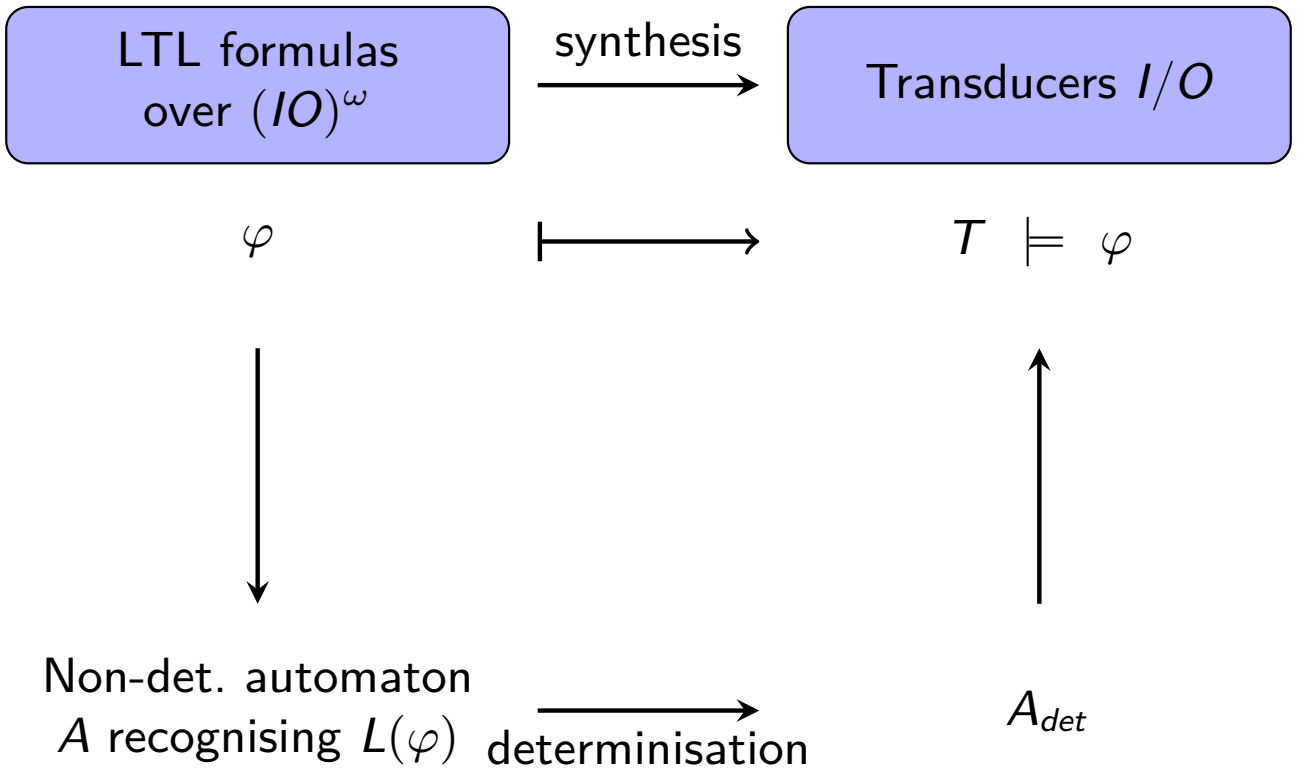

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Problem : this is doubly exponential in the size of φ.

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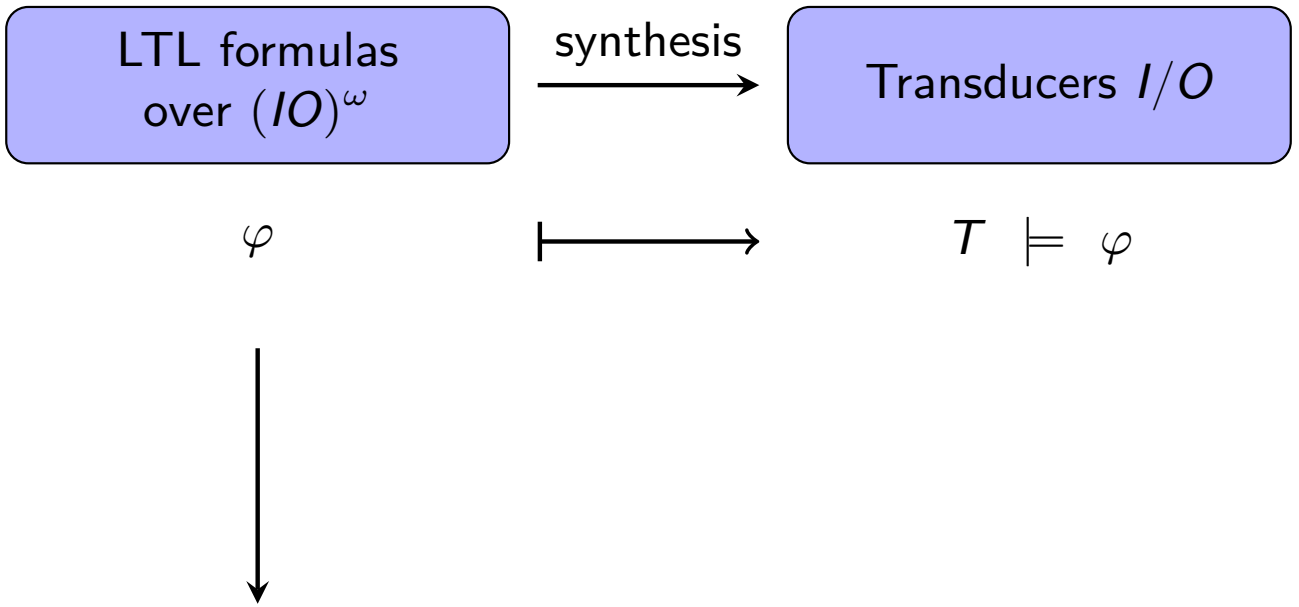

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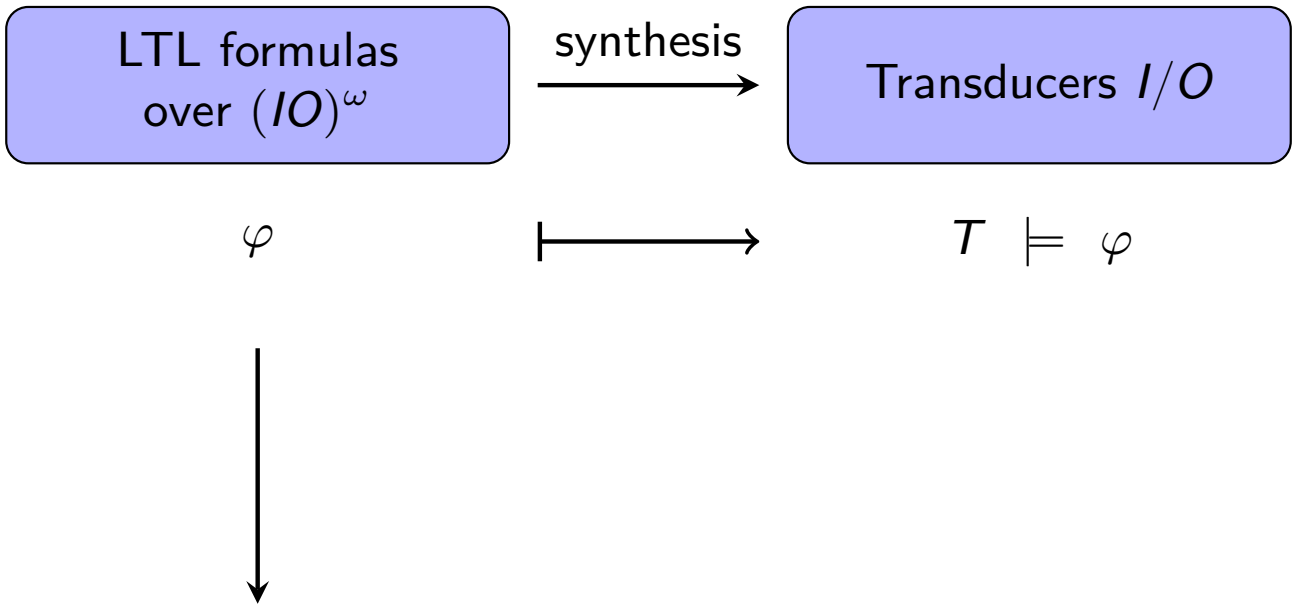

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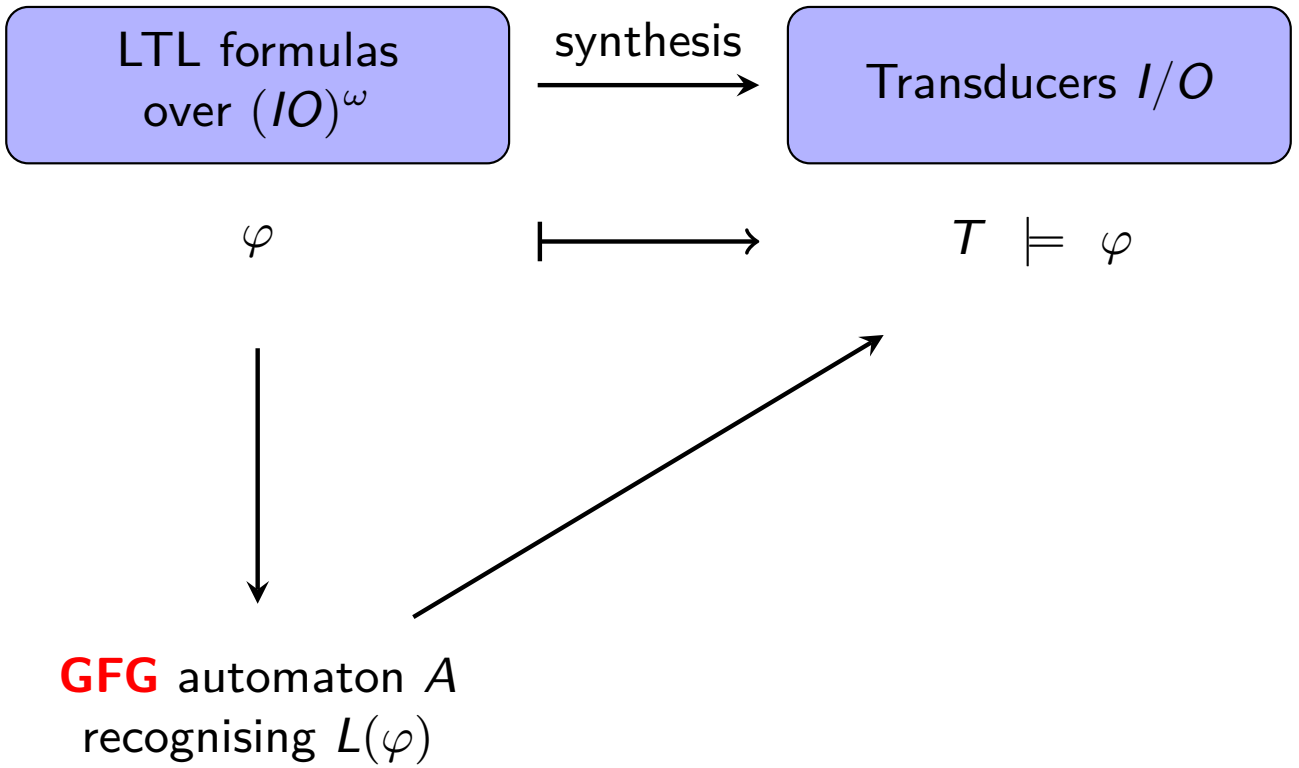

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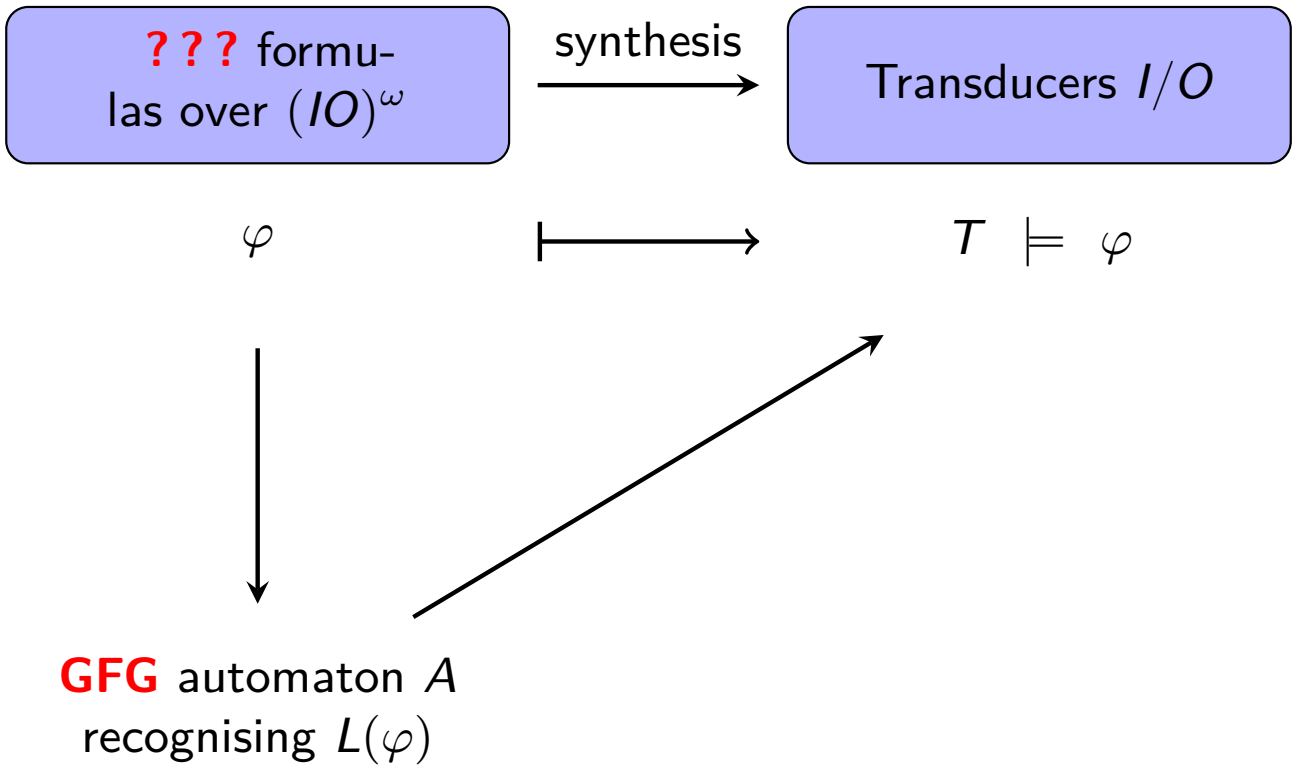

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Let $A = (\Sigma, Q, \Delta, Q_{init}, Acc)$ be a (non-deterministic) automaton for ω-words over Σ, with any kind of acceptance condition.

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Definition (Good-for-Games)

A is said to be **Good-for-Games** (GFG) if there is a strategy σ that resolves the nondeterminism by looking only at the part of the word read so far.

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Run generated by σ over $w = w_1 \dots w_n \dots$:
$\sigma(\epsilon)$
$\sigma(w_1)$
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$\vdots$
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In the context of synthesis :

- We can build a transducer directly from a GFG automaton.
- The strategy σ does **not** need to be known for this algorithm.
- Useful if there are small GFG automata without small deterministic equivalent.

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- The languages K_n are a variation of this first example.

The languages K_n

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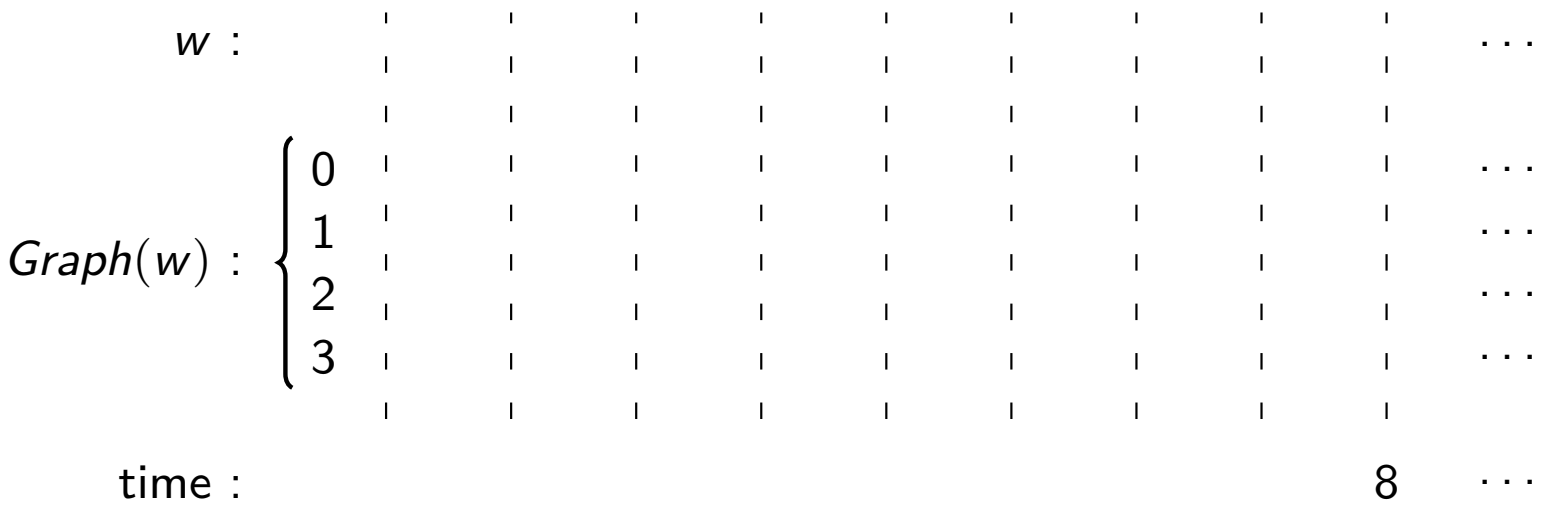

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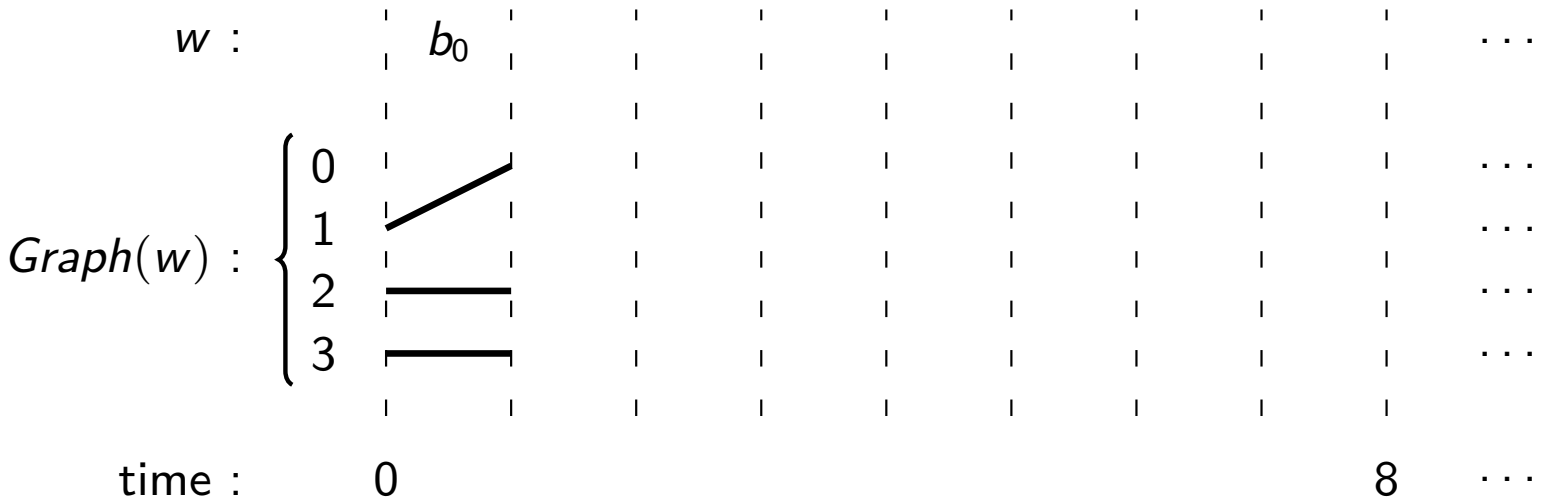

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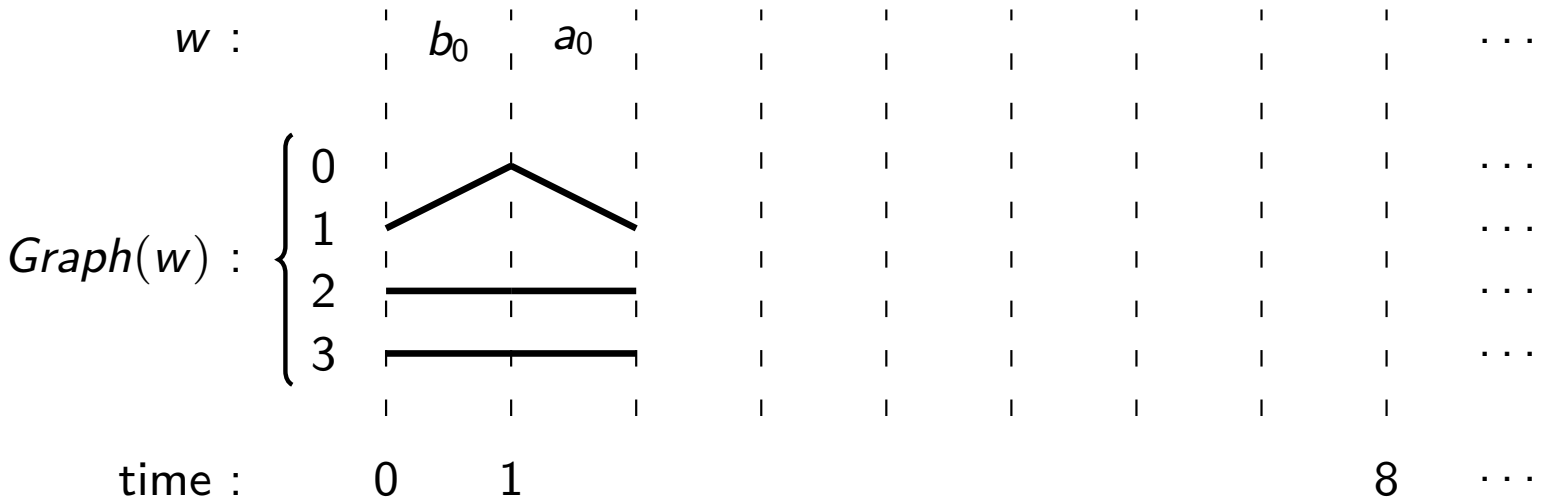

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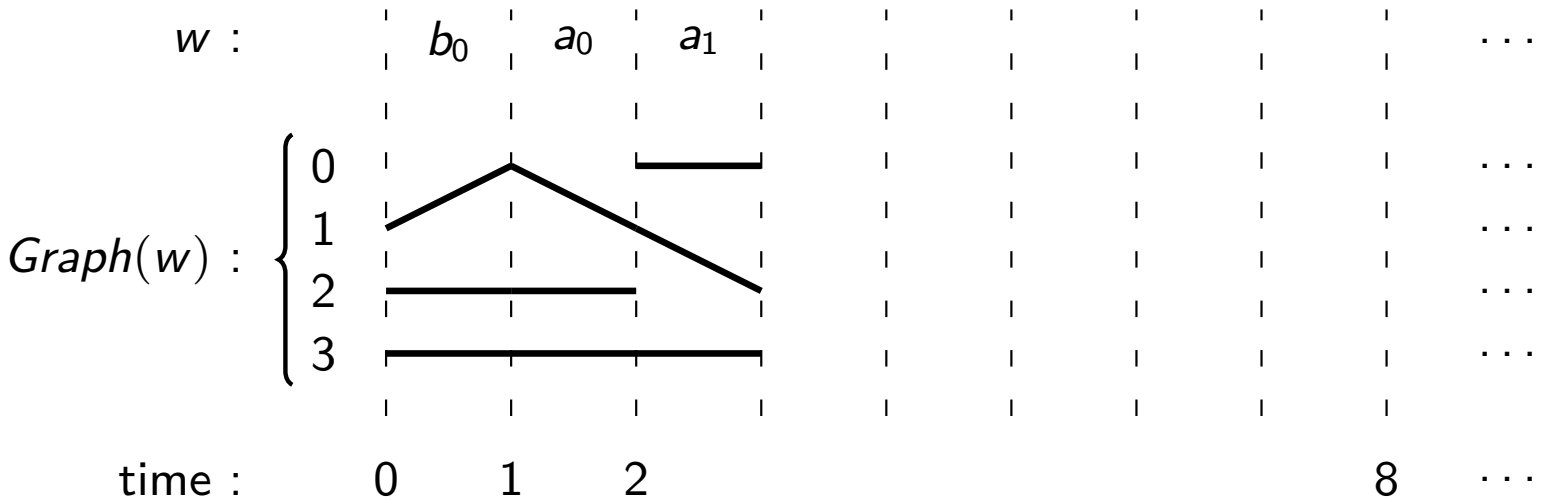

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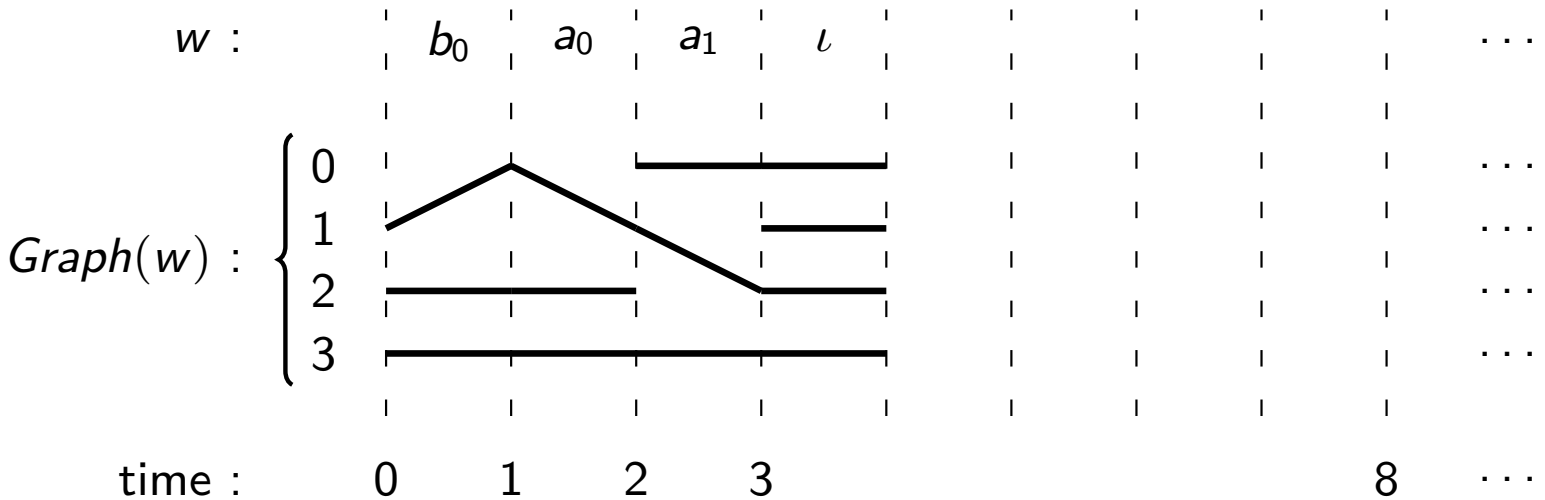

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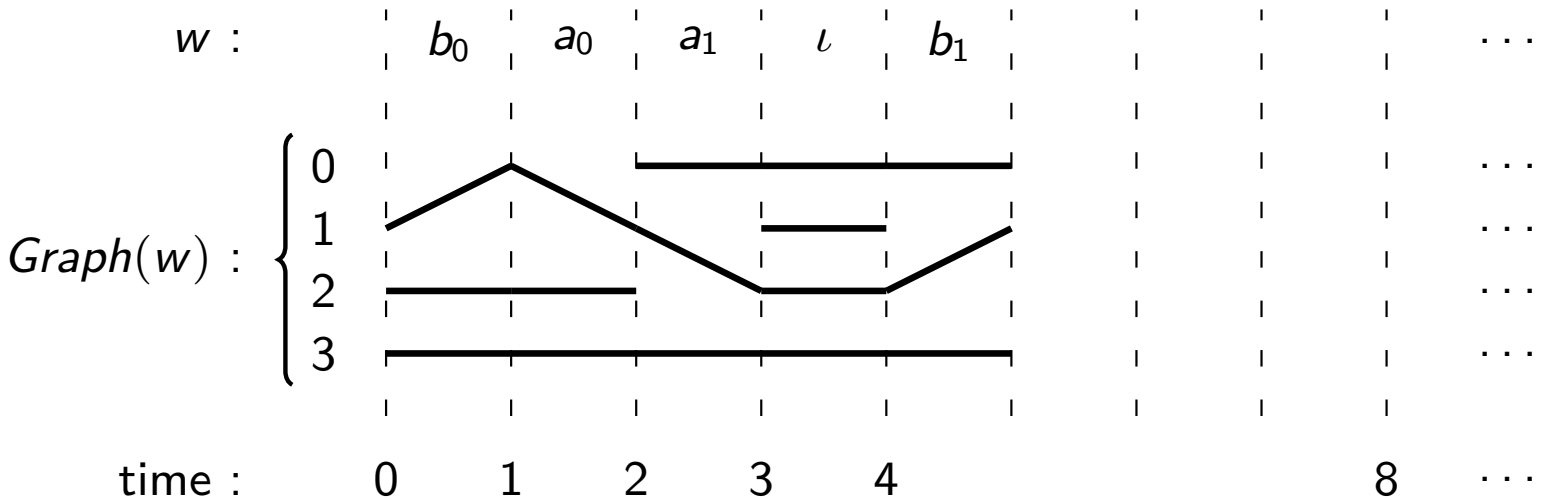

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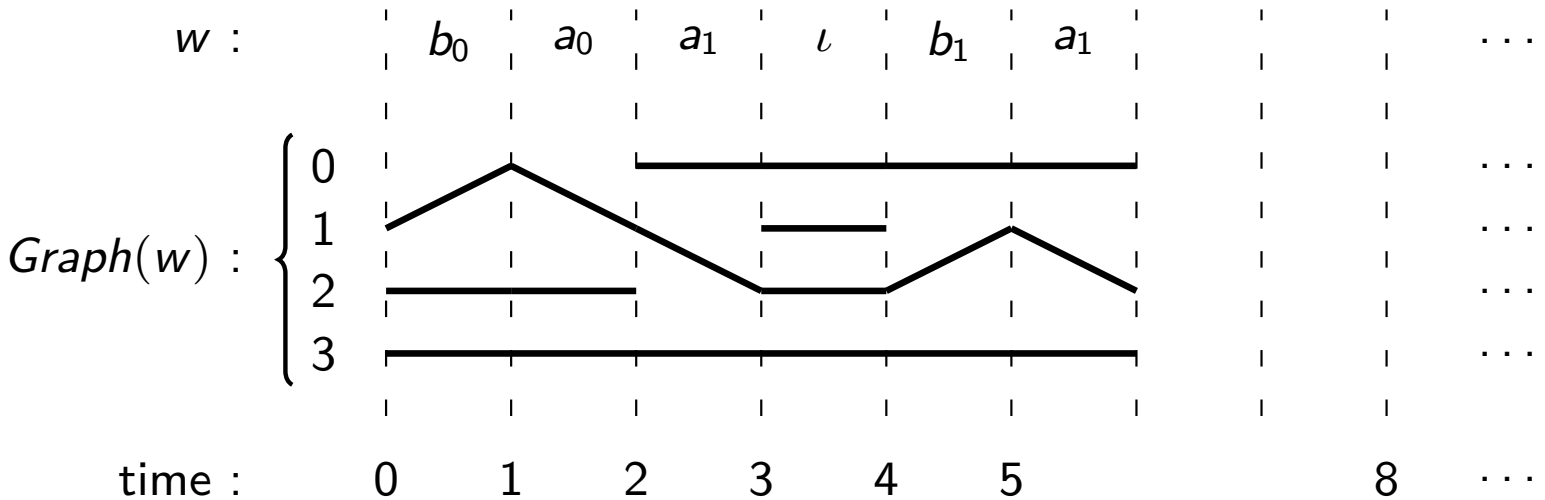

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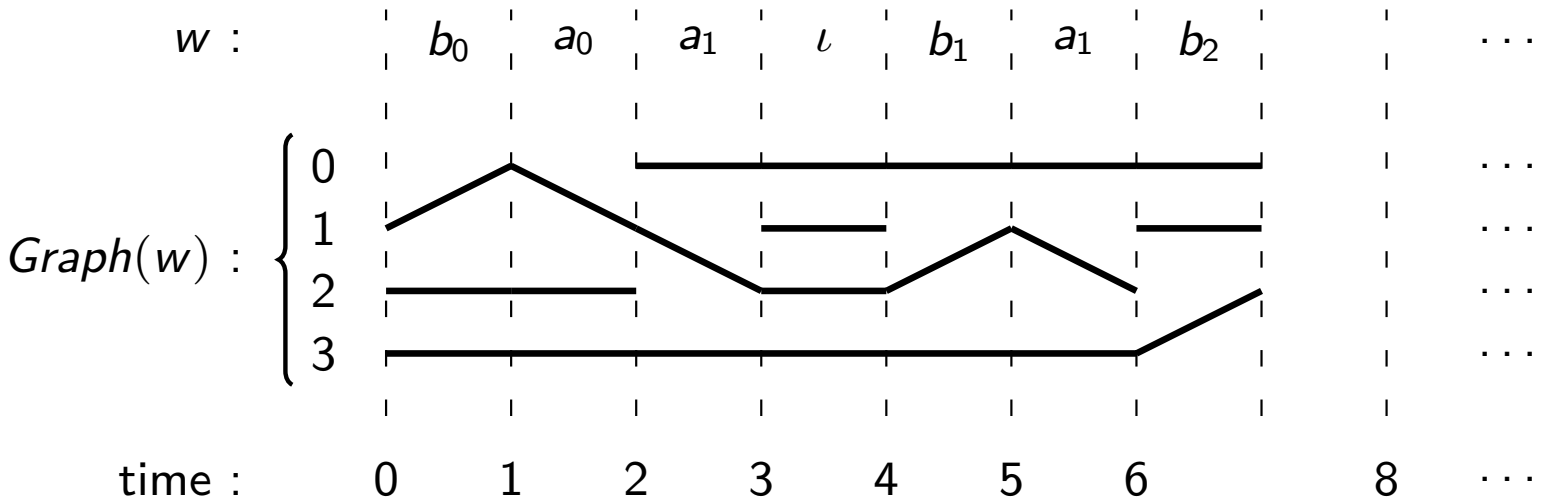

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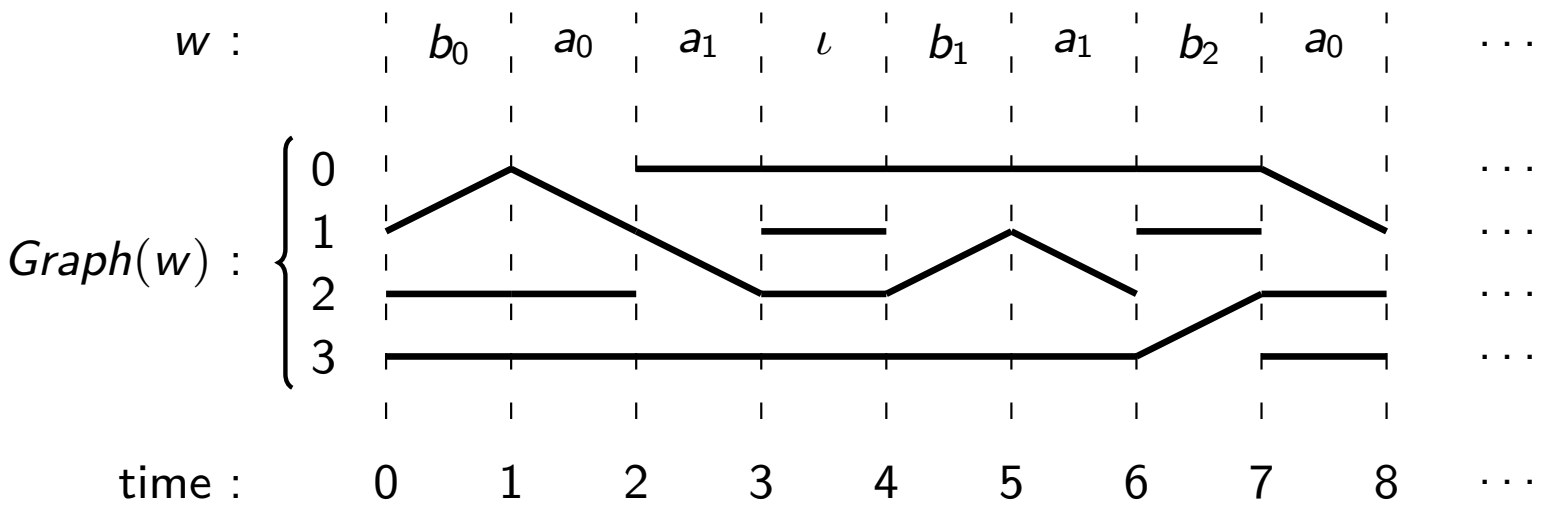

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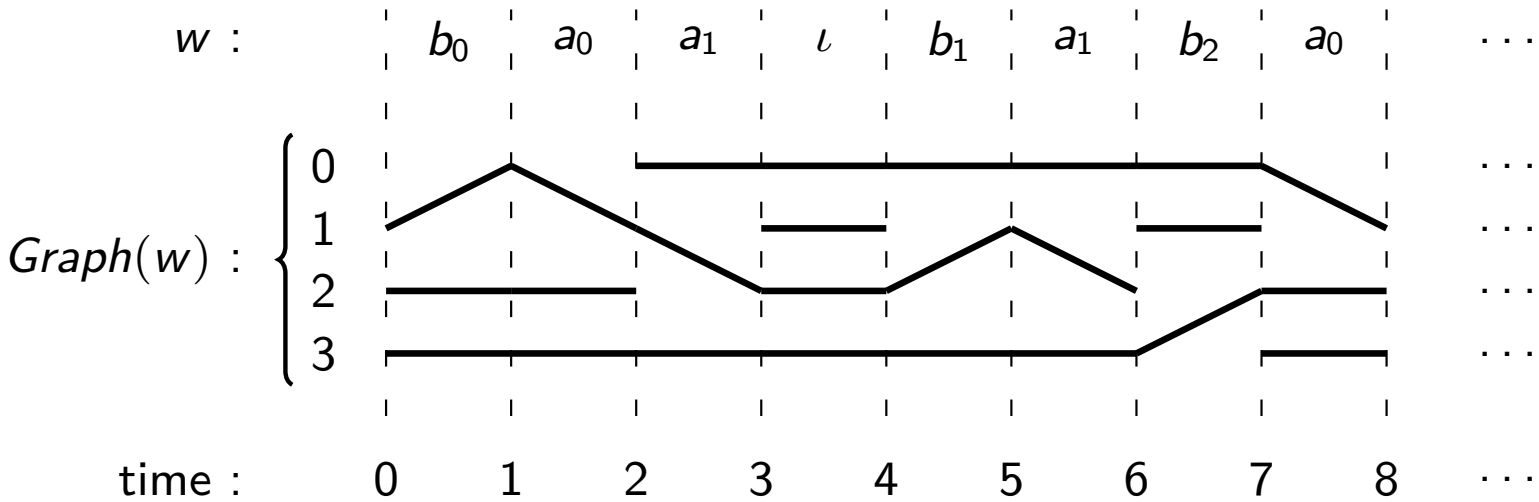
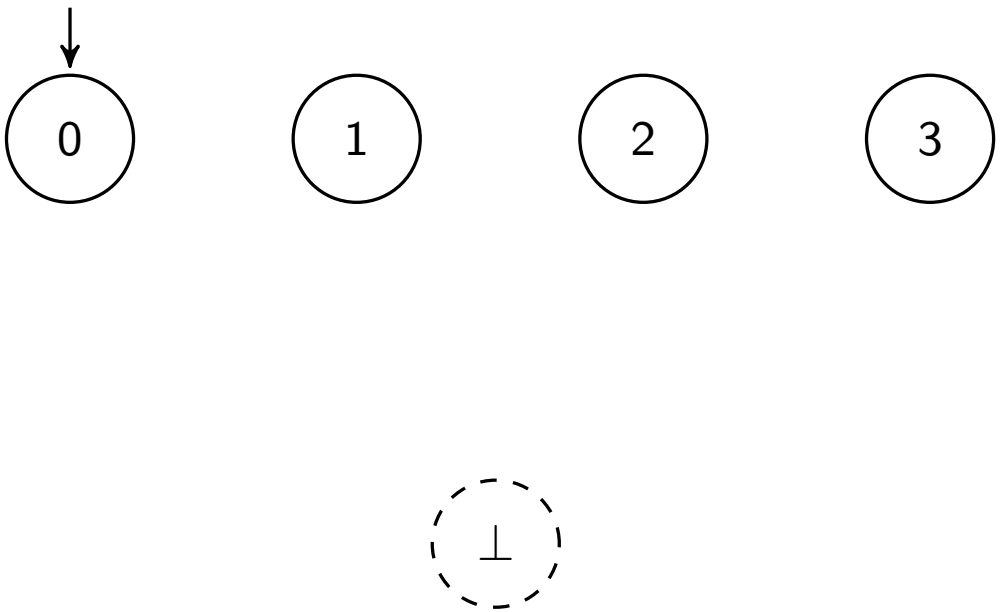


K_n is the set of words w such that $Graph(w)$ has an infinite path.

A GFG coBüchi automaton for K_n :



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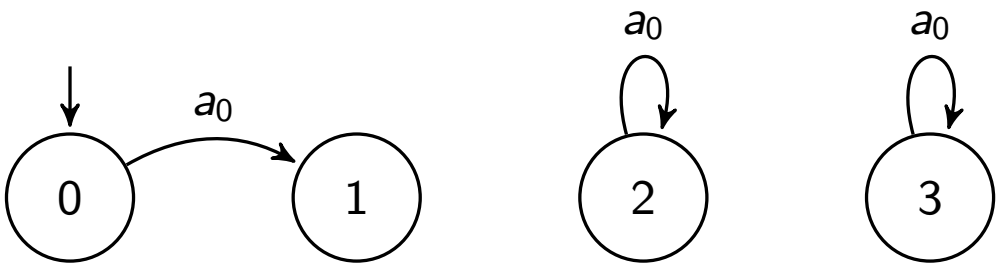

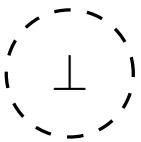

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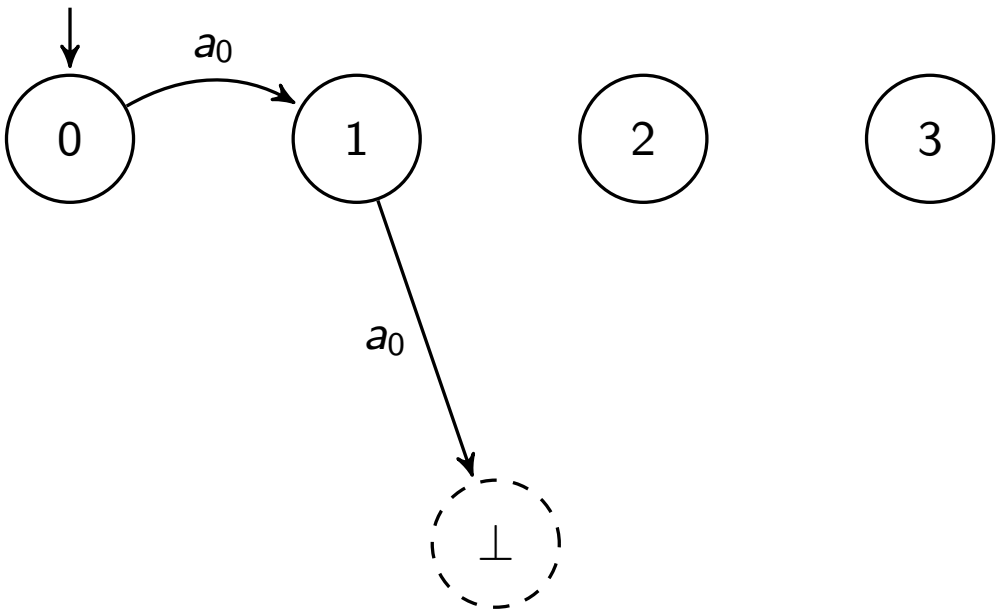

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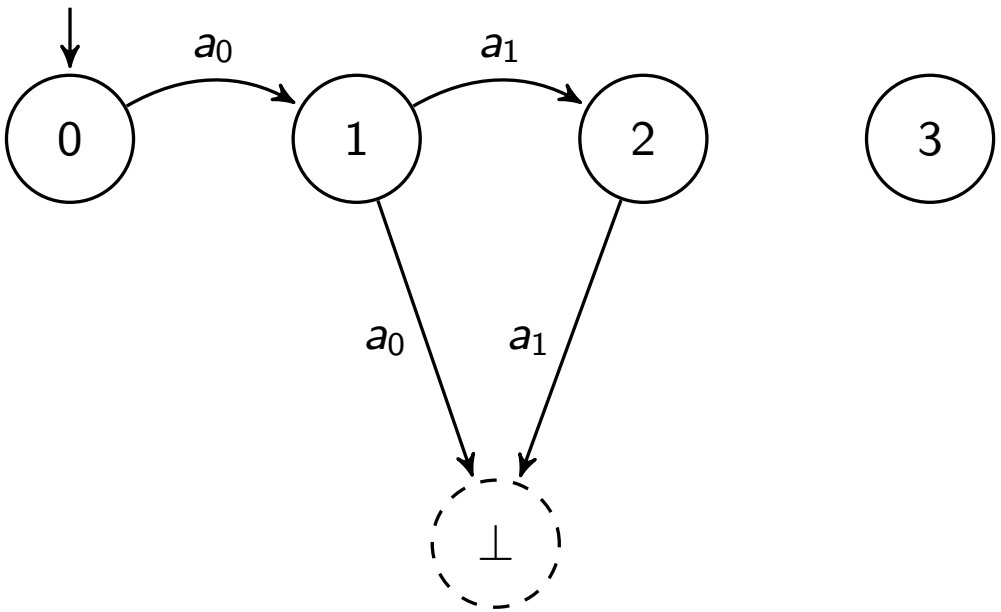

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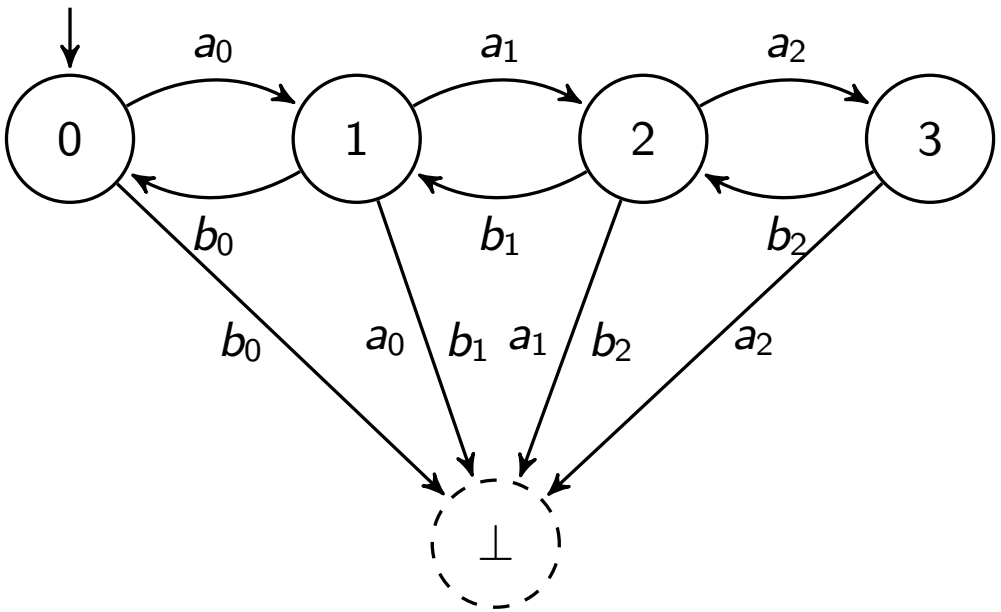

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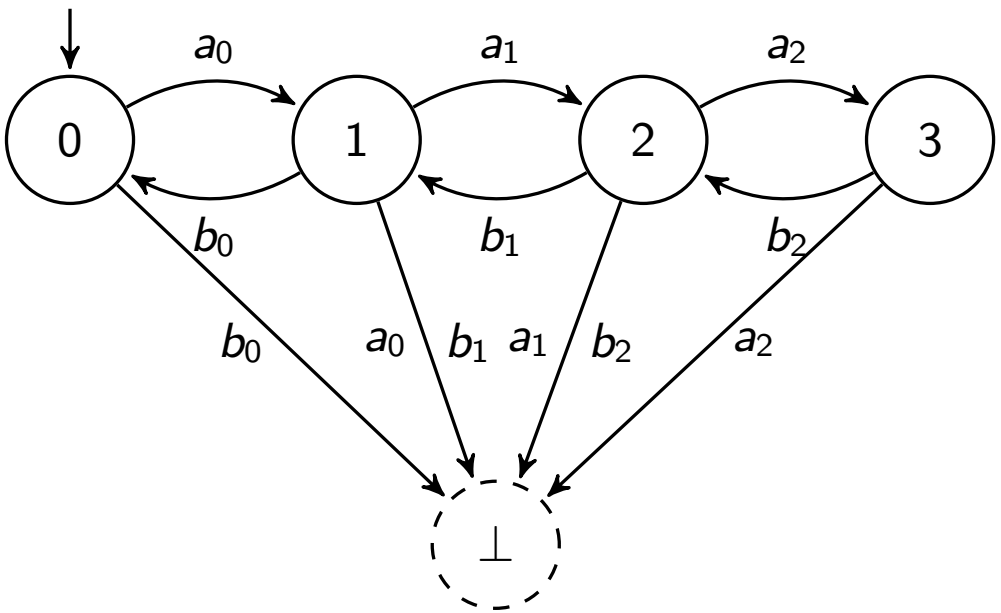

From any letter, jump from $\bot$ to any other state.

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- Problem : The LTL formula we have for K_n has size exponential in n.
- **Conjecture :** Every LTL formula for K_n has size exponential in n.
- **Idea :** Replace LTL with a logic better suited for GFG construction

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Theorem (Folklore ?)

Languages defined by $S\nu TL$ are exactly safety languages.

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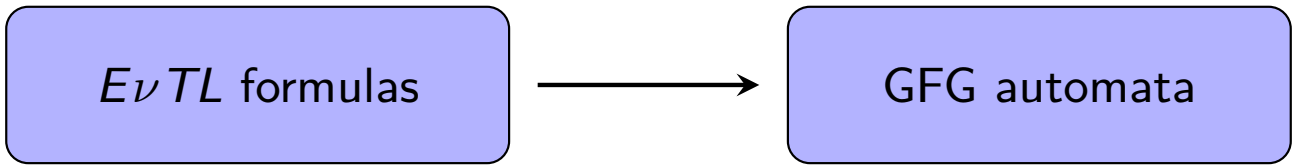
- Can GFG synthesis be done in this context then ?

We can build GFG automata algorithmically from $E\nu TL$ formulas :



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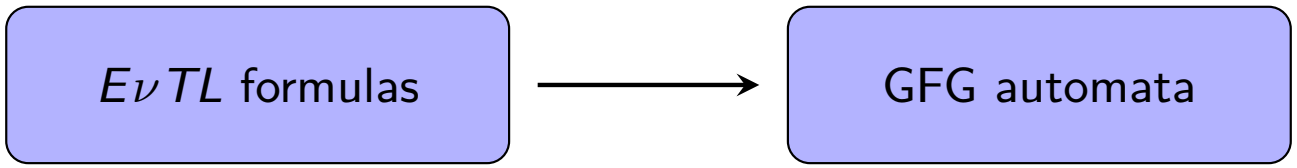

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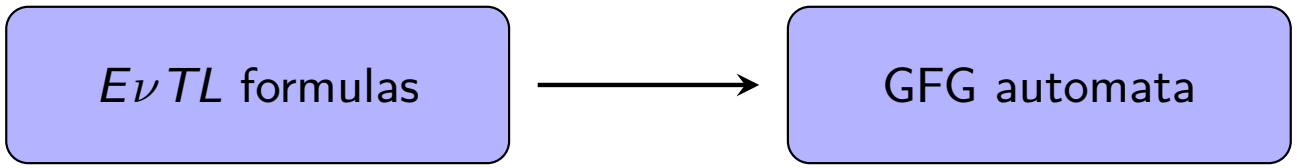

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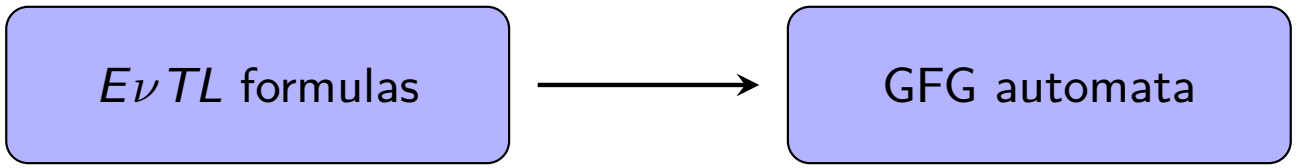

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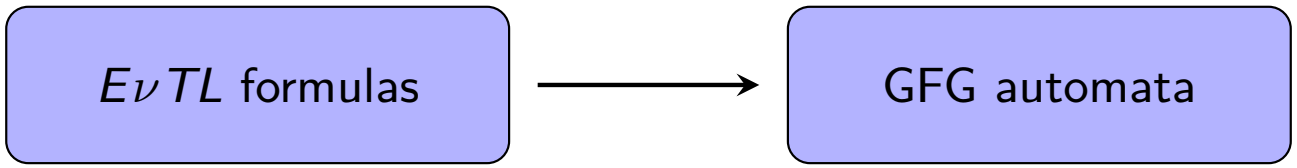

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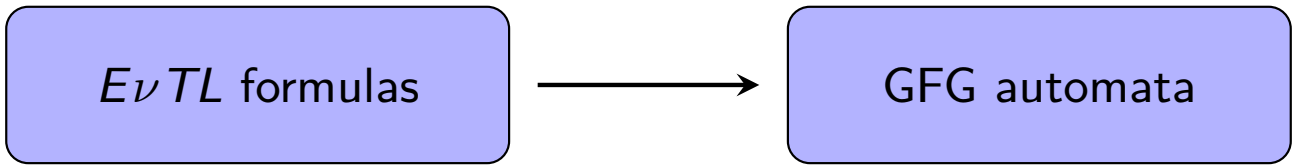

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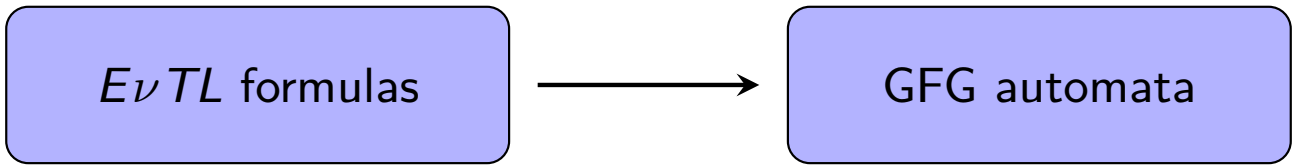

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- See the formula as a $S\nu TL$ formula, and build a safety alternating automaton from it.
- Build an equivalent deterministic safety automaton.
- Minimize the result, and build a coBüchi automaton by adding a $\perp$ state, and following the deterministic transitions when possible, going to $\perp$ otherwise, and jumping to any state from $\perp$.
- GFG-ness follows from the same kind of idea as before : jump to the path that has been uncut for the longest time.

In general : the translation problem from $E\nu TL$ to GFG automata is doubly exponential.

Theorem

The resulting GFG automaton has a minimal amount of states.

Plan

Future work

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- Prove that K_n has no short LTL definition. Would imply succinctness of μ-calculus over LTL !

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- Find fragments of LTL or μ-calculus that are Good-for-Good-for-Games (i.e. no doubly exponential blowup).